# Calculations of Hyperpolarizabilities for Para-disubstituted Benzenes with the QSPR 

Xian Dong ZENG, Xuan XU*, Bing Feng WANG, Bing Can WANG<br>Department of Chemistry, South China Normal University, Guangzhou 510631


#### Abstract

A quantitative structure-property relationship (QSPR) was made for the prediction of the hyperpolarizabilities $(\beta)$ of para-disubstituted benzenes with the nonlinear optical properties, and the $\beta_{\text {cal2 }}$ calculated by this model accorded better with the experimental values ( $\beta_{\text {expl }}$ ) compared with the $\beta_{\text {call }}$ calculated at the $\mathrm{CPHF} / 6-31 \mathrm{G}^{*} / / \mathrm{HF} / \mathrm{STO}-3 \mathrm{G}$ level of theory, especially when $\beta$ was big.


Keywords: QSPR, hyperpolarizabilities, para-disubstituted benzenes, nonlinear optical properties.

When a laser beam propagates through the nonlinear optical (NLO) material, there will be great changes in the properties of the light beam, which are named NLO properties. The NLO properties include the Pockels effect, the second and the third harmonic generation etc. ${ }^{1}$. More generally the NLO material can be used to control the frequency of light, and then it can be used to shorten the wavelength for high-density optical recording and optical switching for optical computing. So the NLO material is regarded as an important material in the information age ${ }^{2}$.

A variety of the microcosmic measurements, such as the dipole moments ( $\mu$ ), polarizabilities $(\alpha)$, hyperpolarizabilities $(\beta)$ and so on, are available for the properties of the nonlinear responses of bulk materials ${ }^{3}$. The predictions of the $\mu, \alpha$ can be done very well by $a b$ initio method, but the prediction of $\beta$ is still not satisfactory. It is well known that the properties of a compound are determined by its microstructure, but little attention has been put on the statistical method - quantitative structure -property relationship (QSPR) which can be used to improve the precision of the same series compounds by theoretical calculation. The electronic structure and the $\alpha, \beta$ of 32 para-disubstituted benzenes with the NLO properties were calculated by coupled perturbed Hartree-Fock (CPHF) method at the 6-31G* level of theory.

## Methodology

The electronic structures were optimized by HF/STO-3G method after primarily optimized by PM3 method.

The components of the molecular hyperpolarizability tensors $\left(\beta_{i j k}\right)$ were obtained by means of the CPHF at the $6-31 \mathrm{G}^{*}$ level ${ }^{4}$. $\beta_{\mu}$ is its projection on the direction of dipole

[^0]moment. To keep the comparability with the experimental results and the results from semiempirical method in the reference ${ }^{5}, \beta_{\mu}$ was calculated using the following expression:
\[

$$
\begin{aligned}
& \beta_{\mu}=(3 / 5)\left(\beta_{x} \mu_{x}+\beta_{y} \mu_{y}+\beta_{z} \mu_{z}\right) /\left(\mu_{x}^{2}+\mu_{y}^{2}+\mu_{z}^{2}\right)^{1 / 2} \\
& \beta_{i}=\sum_{j} \beta_{i j j} \quad i, j=(x, y, z)
\end{aligned}
$$
\]

A great variety of the quantum chemical indices were taken from the calculation results, such as the lowest unoccupied molecular orbital energy $E_{\text {LUMO }}$, the highest occupied molecular orbital energy $E_{\text {Номо }}, D E L H=E_{\mathrm{LUMO}}-E_{\text {Номо }}$, total energy $E, \mu$ and solvent-excluded volume $V$ etc.. Based on these quantum chemical indices, the stepwise regression analysis was used to build a quantitative structure-property relationship model, the regression function (1), for hyperpolarizability of para- disubstituted benzenes

## Results and Disscussion

Table 1 Some of the quantum chemical indices of studied para-disubstituted benzenes*

| NO. | . X | Y | Debye | DELH / <br> Hartree | $\mathrm{V} / \AA^{3}$ | NO. | X | Y | $\begin{gathered} \mu / \\ \text { Debye } \end{gathered}$ | DELH / <br> Hartree | $\mathrm{V} / \AA^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{SO}_{2} \mathrm{Me}$ | OH | 3.9 | 0.44505 | 127.624 | 17 | NO | $\mathrm{NMe}_{2}$ | 4.5 | 0.36836 | 126.917 |
| 2 | $\mathrm{SO}_{2} \mathrm{C}_{3} \mathrm{~F}_{7}$ | OMe | 4.9 | 0.41360 | 210.668 | 18 | $\mathrm{NO}_{2}$ | Me | 4.6 | 0.40582 | 107.028 |
| 3 | CN | Me | 4.0 | 0.43525 | 100.544 | 19 N | $\mathrm{NO}_{2}$ | Br | 2.9 | 0.39377 | 110.429 |
| 4 | CN | Cl | 2.0 | 0.43095 | 98.625 | 20 N | $\mathrm{NO}_{2}$ | OH | 4.2 | 0.39939 | 94.769 |
| 5 | CN | Br | 2.3 | 0.42193 | 103.951 | 21 N | $\mathrm{NO}_{2}$ | OPh | 4.7 | 0.36448 | 159.08 |
| 6 | CN | OPh | 4.0 | 0.40517 | 152.558 | 22 | $\mathrm{NO}_{2}$ | OMe | 4.7 | 0.39626 | 113.845 |
| 7 | CN | OMe | 4.1 | 0.43135 | 107.351 | 23 | $\mathrm{NO}_{2}$ | SMe | 4.5 | 0.36744 | 125.234 |
| 8 | CN | SMe | 3.8 | 0.40223 | 118.723 |  | $\mathrm{NO}_{2}$ | $\mathrm{N}_{2} \mathrm{H}_{3}$ | 5.6 | 0.38397 | 107.099 |
| 9 | CN | $\mathrm{NH}_{2}$ | 5.2 | 0.41751 | 90.622 | 25 N | $\mathrm{NO}_{2}$ | $\mathrm{NH}_{2}$ | 5.8 | 0.37721 | 97.123 |
| 10 | CN | $\mathrm{NMe}_{2}$ | 5.4 | 0.40686 | 127.732 | 26 | $\mathrm{NO}_{2}$ | $\mathrm{NMe}_{2}$ | 6.1 | 0.36593 | 134.226 |
| 11 | CHO | Me | 2.9 | 0.42345 | 103.81 | 27 N | $\mathrm{NO}_{2}$ | CN | 0.6 | 0.4007 | 103.781 |
| 12 | CHO | OPh | 3.3 | 0.40487 | 158.952 | 28 N | $\mathrm{NO}_{2}$ | CHO | 2.7 | 0.39311 | 107.034 |
| 13 | CHO | OMe | 3.6 | 0.41862 | 110.624 | 29 | $\mathrm{CHC}(\mathrm{CN})$ | OMe | 6.3 | 0.36294 | 145.138 |
| 14 | CHO | SMe | 3.4 | 0.39108 | 122.02 |  | $\mathrm{CHC}(\mathrm{CN})$ | $\mathrm{NMe}_{2}$ | 6.8 | 0.34622 | 166.173 |
| 15 | CHO | $\mathrm{NMe}_{2}$ | 4.2 | 0.39174 | 131.033 |  | $\mathrm{C}_{2}(\mathrm{CN})_{3}$ | $\mathrm{NH}_{2}$ | 7.4 | 0.31999 | 142.395 |
| 16 | $\mathrm{COCF}_{3}$ | OMe | 3.9 | 0.40327 | 209.456 |  | $\mathrm{C}_{2}(\mathrm{CN})_{3}$ | $\mathrm{NMe}_{2}$ | 7.9 | 0.31059 | 179.258 |



Table 1 gave some of the quantum chemical indices introduced in the regression function that was given as below:

$$
\begin{aligned}
& \beta=36.9800+0.01165 \exp (\mu)-87.4506 D E L H+0.005094 V \\
& \mathrm{~N}=32, \mathrm{R}=0.974, \mathrm{SD}=2.755, \mathrm{~F}=169.593 \\
& t_{\exp (\mu)}=11.216, t_{D E L H}=-3.441, t_{V}=0.295, r_{\exp (\mu)}=0.961, r_{\text {DELH }}=-0.841, r_{V}=0.387
\end{aligned}
$$

Where the correlation coefficient R was 0.974 , the standard deviation SD was 2.755 , and F-test value was 169.593 . The QSAR model had good predictability. Eq. (1) showed that $\beta$ was a positive correlation to $\exp (\mu)$. The value of $r_{\exp (\mu)}$, the single correlation coefficient $\exp (\mu)$ to $\beta$ reached 0.961 , and $t_{\exp (\mu)}$ was 11.216 . It meant that higher $\mu$ leaded to higher $\beta$, and the subtle change of $\mu$ would affect $\beta$ greatly, that is, $\mu$ was the primary factor. It also showed that the influence of $D E L H$ cannot be ignored, and the $r_{D E L H}$ was 0.841. The molecule with small energy difference between the $H O M O$ and the $L U M O$, $D E L H$, was easily energized, and the modulus of $D E L H$ in Eq. (1) showed that smaller $D E L H$ leaded to bigger $\beta$, too. The molar volume $V$ was introduced into Eq. (1) as a positive correlation factor. For para-disubstituted benzenes, the volume of molecules and the degree of delocalization increased with the increasing volume of substituents which contain the $p_{\mathrm{z}}$ orbital, which accorded with the relationship between the $\beta$ and the $\pi$-conjugated condition as known $^{3} . \beta$ increased with the increasing degree of delocalization. Table 2 gave concrete data for the following comparison, and the regression equations can be derived from it as below:

$$
\begin{gathered}
\beta_{\text {expt }}=2.85( \pm 1.21)+0.64( \pm 0.08) \beta_{\text {call }} \\
\mathrm{R}=0.792, \mathrm{SD}=5.7, \mathrm{~F}=50.57, \mathrm{~N}=32 \\
\beta_{\text {expt }}=0.38( \pm 0.55)+0.95( \pm 0.04) \beta_{\text {cal2 }} \\
\mathrm{R}=0.948, \mathrm{SD}=2.6, \mathrm{~F}=545.12, \mathrm{~N}=32
\end{gathered}
$$

Both $\mu$ and $\alpha$ can be accurately calculated by the $a b$ initio, and they fitted the experimental values better than those calculated using MNDO/PM3 method by Matsuzawa et. al. ${ }^{5}$. Table 2 showed that our theoretical prediction of $\beta\left(\beta_{\text {call }}\right)$ was still not satisfactory, especially when $\beta$ was large. Comparing Eq. (2) with Eq. (3), R increased from 0.792 to $0.948, \mathrm{~F}$ value raised sharply from 50.57 to 545.12 , and the SD in Eq. (2) was bigger than that in Eq. (3). In Eq. (3), the coefficient of $\beta_{\text {cal2 }}$ was 0.95 , and the constant was small. The differences were also indicated in Figure 1 clearly.

Table 2 Comparison of calculated results with experimental values a


[^1]The regression line $l_{2}$ of Eq. (3) in Figure 1 almost overlapped the dash line $y=x$, and the sample points uniformly distributed both sides of it, which meant systematic error was quite low. On the contrary, the regression line $l_{1}$ of Eq. (2) in Figure 1 deviated from the dash line $y=x$, which meant that the theoretical predictions of $\beta$ were prevalent smaller than experimental values, a systematic error existed in the theoretical method at present, and the prediction ability was limited for the compounds with great $\beta$. The comparison showed the statistical method-QSPR predict the hyperpolarizabilities $\beta$ accurately for the series of para-disubstituted benzenes.

Figure 1 The plots of the Hyperpolarizabilities calculated by theoretical method and QSPR $v s$. the observed values


## References

1. S. M. Liu, J. J. Xu, R. Guo. Coherent Optics Theorems and Appliance, Nankai University Press, Tianjing: 2001, pp.146-155, pp.246-311.
2. S. R. Marder, B. Kippelen, A. K. Y. Jen et al., Nature, 1997, 388, 345.
3. S. R. Marder, C. B. Gorman, F. Meyers et al., Science, 1994, 265, 632.
4. A. D. Buckingham. Adv. Chem. Phys., 1967, 12, 107.
5. N. Matsuzawa, D. A. Dixon. J. Phys. Chem., 1992, 96, 6232.

Received 20 May, 2003


[^0]:    * E-mail: xuxuan@scnu.edu.cn

[^1]:    a. Experimental data and the value marked cal were both taken from Ref.5;
    b. Calculated by CPHF/6-31G* method;
    c. Calculated by QSPR model.

